Jake Traut

3/20/2017

CSCI 4446 Problem Set 9

1. Gathering data by running RK4 on Lorenz system
2. Wolf Algorithm for determining Lyapunov exponents from data

The Lyapunov exponent characterizes the rate of separation of compact trajectories, as seen in the Lorenz system. Our goal was to implement Wolf’s algorithm to compute the biggest (positive) Lyapunov exponent on two sets of data from the Lorenz system; one being chaotic and the other non-chaotic. My lambda1 result (the biggest Lyapunov exponent) for the chaotic data set came out to be **2.1444** which is actually quite close to the value I was expecting (around 2.16) based off of Wolf’s paper Physica 16D, 285 (1985) published calculation, which used an r value closer to 46. This number makes sense as the Lorenz system overall remains fairly tight throughout its chaotic trajectory so even the biggest Lyapunov exponent remains small in the rate of separation.

As for the non-chaotic data set my implementation of Wolf’s algorithm returned a lambda value of 0. This actually makes sense though as the Lorenz attractor with r = 18 (the r value I used with RK4) follows a long curve from the starting point to a single attractor where it makes it’s only spiral inward, where the “fiduciary” trajectory is mainly that spiral and the z trajectory where neighbors are being searched for being the that long curve that eventually makes its way into the spiral. I don’t think Wolf’s algorithm is meant for non-chaotic data.

4. Calculating Lyapunov exponents with system derivative, variational equations, and eigenvalue computations.

Using the equation given on the 10000th integration run of the variational equations, I got a lambda1 (the biggest positive Lyapunov exponent) value of 1.348. This is not what I was expecting as I believe my original value produced by Wolf’s algorithm was accurate based upon the published calculations of the value and much effort going into the implementation, and this output is much less in comparison. But thinking about it I don’t feel this method is producing a creditable value as we are approximating with the 10000th step over a 0.001s timestep, which is only a total of 10 seconds into the trajectory when the true value wants the limit as time goes to infinity, and 10 seconds is not nearly close to infinite time.